Measures of acceleration in cosmology

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Outline

1 Evidence for an accelerating Universe

2 Inhomogeneity and the Fitting Problem

3 Acceleration measures

4 Application to inhomogeneous models

Evidence for an accelerating universe

Data-driven cosmology

Cosmology is awash with good quality data

CMB anisotropies (WMAP, QUIET, ACT, Planck ...)

Type Ia supernovae (SNLS, SDSS, DES ...)

Galaxy redshift surveys (SDSS, WiggleZ ...)

Well-developed theory

Highly-developed theoretical model

FLRW spacetime with perturbations

Linear regime well-understood

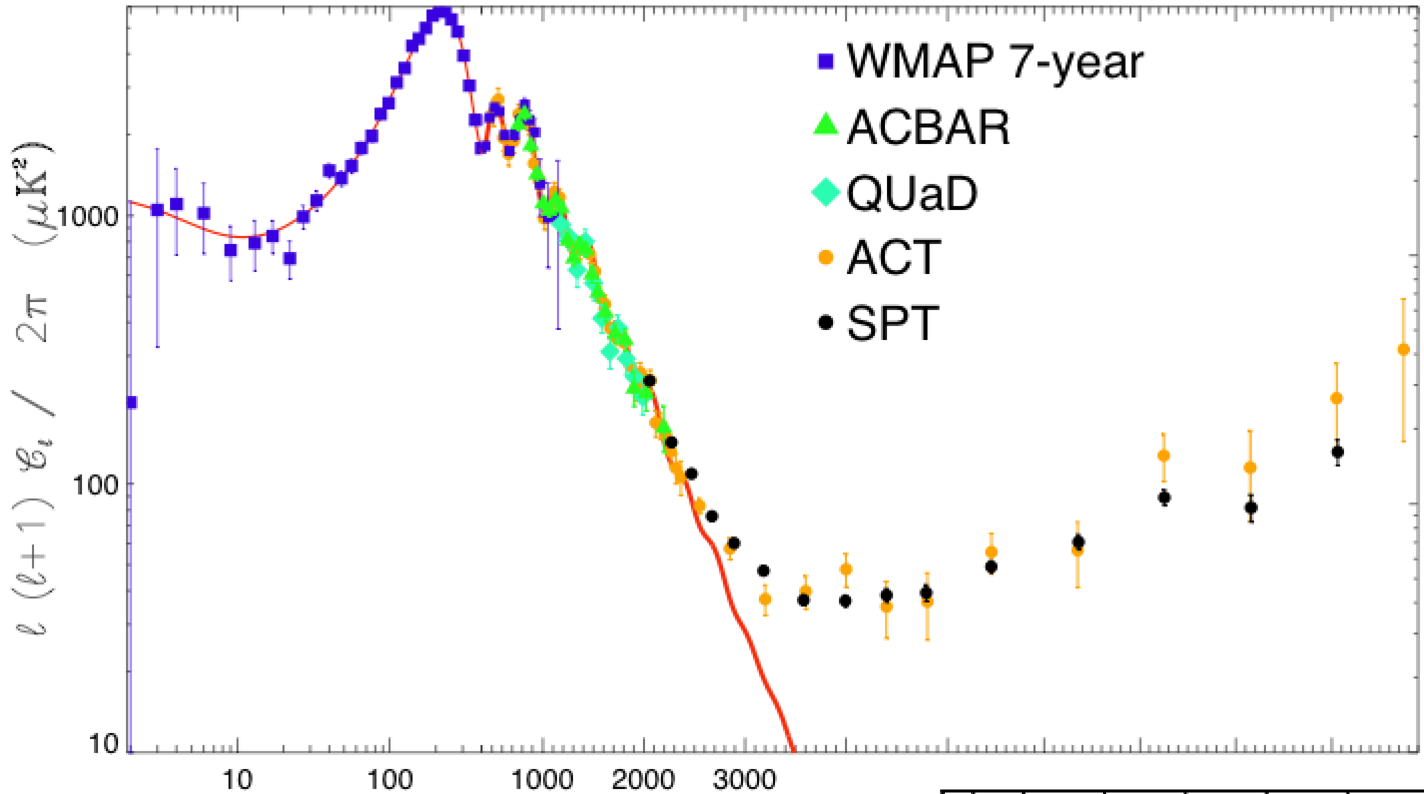
Mature computer codes

Theory fits the data

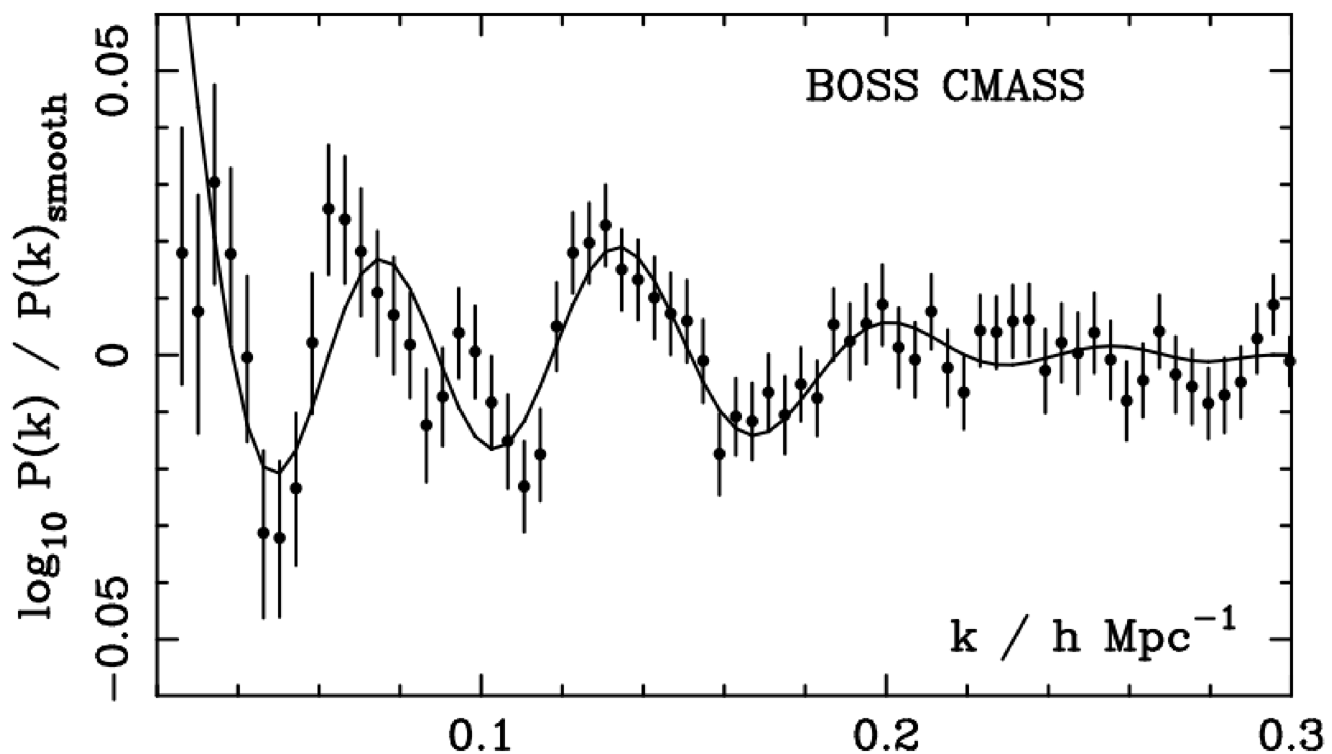
Theoretical model predictions match available data beautifully

Dark matter + dark energy required

ΛCDM: “Concordance” model



CMB Power Spectrum (Shirokoff et al. 2010)



Baryon Acoustic Oscillations (Anderson et al. 2012)

Problems with ΛCDM

Numerous puzzles (mostly about dark energy)
e.g. cosmological constant problem, coincidence problem, ...

ΛCDM is a **phenomenological model**

Underlying physics is **not** understood

ΛCDM as a model of spacetime

What does the phenomenological model actually tell us about spacetime?

The real spacetime is inhomogeneous

ΛCDM parameters (H_0, Ω_m, ...) denote "average" or "typical" properties

Inhomogeneity and the Fitting Problem

Fitting Problem

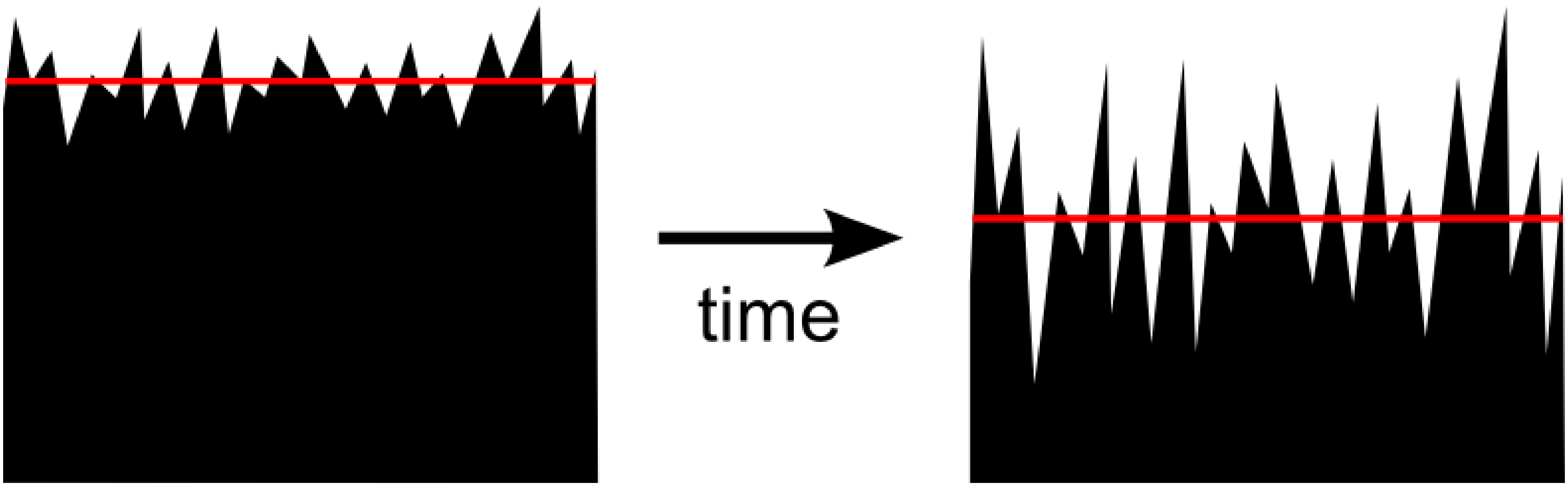
Want to fit homogeneous, isotropic model to **observations** of real "lumpy" Universe

There is no **real** FLRW spacetime, just some idealised theoretical entity

Ellis & Stoeger (1987): What fitting procedure makes the most sense?

Averaging Problem

But also want a model that tracks the average **evolution** of the spacetime



Averaging Problem

Spatial averaging procedure is not well defined

Scalar averaging? Covariant averaging?

Review by van den Hoogen (arXiv:1003.4020)

Model mismatch

In general, the model that fits the observations need not be the same as the one that describes the average evolution

Models of backreaction invoked to explain dark energy

Review by Räsänen (arXiv:1102.0408)

Key question

What is meant by "acceleration" in an inhomogeneous universe?

Clearly, there are different types

Acceleration measures

Types of acceleration

Define possible measures of acceleration, based on observational and/or theoretical procedures

Relate measures to observables and/or underlying properties of spacetime

Deceleration parameters

Convenient to re-use “deceleration parameter” notation from FLRW models

$$a(t) = 1 + H_0\,(t - t_0) - \frac{1}{2}\,q_0\,H_0^2\,(t - t_0)^2 + \ldots$$

$$q(t) = -\frac{1}{a}\,\frac{\mathrm{d}^2 a}{\mathrm{d}t^2}\,\left[\frac{1}{a}\,\frac{\mathrm{d}a}{\mathrm{d}t}\right]^{-2}$$

$q < 0$ means acceleration

Local volume acceleration

3+1 decomposition of spacetime;
write down Einstein equations, e.g.

$$\dot{\Theta} = -\frac{1}{3}\Theta^2 - \frac{1}{2}(\rho + 3p) + \Lambda - 2\sigma^2$$

“Local volume” deceleration parameter:

$$q_\Theta = -1 - 3\frac{\dot{\Theta}}{\Theta^2} = \frac{3}{\Theta^2}\left[\frac{1}{2}(\rho + 3p) - \Lambda + 2\sigma^2\right]$$

Local volume acceleration

Determines whether the expansion of spacetime itself is accelerating

Acceleration can vary from place to place

For acceleration, need dark energy or cosmological constant

Hubble diagram acceleration

Fit FLRW distance-redshift relation to observations

$$d_A(z) = \frac{cz}{H_0}\left(1 - \frac{1}{2}(3 + q_0)z + \mathcal{O}(z^2)\right)$$

Deceleration parameter in fitted model:

$$q_0 = -\left.\frac{d''_A}{d'_A}\right|_0 - 3$$

Hubble diagram acceleration

Corresponds to what observers actually do with supernova data

Defined in any spacetime, but must deal with anisotropies in d(z)

Need to solve null geodesic equations (difficult in general)

Spatial average acceleration

Choose a foliation of spacetime and average scalars over spacelike domain

$$\langle S \rangle = V_{\mathcal{D}}^{-1} \int_{\mathcal{D}} S(\vec{x}, t) \sqrt{-h}\, d^3x$$

Use spatial volume to define a scale factor in "spatial average" model

$$V_{\mathcal{D}} = \int_{\mathcal{D}} \sqrt{-h}\, d^3x \qquad a_{\mathcal{D}}(t) = \left(\frac{V_{\mathcal{D}}(t)}{V_{\mathcal{D}}(t_0)} \right)^{\frac{1}{3}}$$

Spatial average acceleration

Write down evolution equations for "spatial average" model (Buchert 2000, Marochnik 1975)

$$3H_{\mathcal{D}}^2 \;=\; 8\pi G\langle\rho\rangle + \Lambda - \frac{1}{2}\left(Q_{\mathcal{D}} + \langle {}^{(3)}\mathcal{R}\rangle\right)$$

$$3\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} \;=\; -8\pi G\langle\rho\rangle + \Lambda + Q_{\mathcal{D}}$$

Get extra "backreaction" term because spatial averaging and time evolution don't commute

$$Q_{\mathcal{D}} = \frac{2}{3}\left(\langle\Theta^2\rangle - \langle\Theta\rangle^2\right) - 2\langle\sigma^2\rangle$$

Spatial average acceleration

Define deceleration parameter:

$$q_{\mathcal{D}} = -\frac{1}{H_{\mathcal{D}}^2}\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}$$

Condition for acceleration now also depends on backreaction term

Kristian-Sachs local observables

Generalised local series expansion of distance-redshift relation (Kristian & Sachs 1966)

$$d_A = \frac{z}{[K^a K^b \nabla_a u_b]_0} \left(1 - \left[\frac{K^a K^b K^c \nabla_a \nabla_b u_c}{2(K^d K^e \nabla_d u_e)^2} \right]_0 z + \mathcal{O}(z^2) \right)$$

$$K^a = \frac{k^a}{[u_b k^b]_0} = -u^a + e^a$$

Define deceleration parameter:

$$q_{\mathrm{KS}} = \frac{3}{\Theta^2} \left[\frac{1}{2}(\rho + 3p) - \Lambda + 6\sigma^2 \right]_0$$

Summary of measures

1. **“Local”:** Raychaudhuri equation
2. **“Observational”:** from Hubble diagram
3. **“Average”:** Spatial average (scalar averaging)
4. **“Kristian-Sachs”:** Local observables

Inhomogeneous models

Statistically-homogeneous spacetimes

Spherical collapse model

Disjoint FLRW models

Simple, intuitive toy model

1) Expanding vacuum region (voids)
2) Collapsing dust region (overdensities)

Set $\Lambda = 0$ in all that follows

Geometry of system

Disjoint; no need to specify global arrangement of regions

But need this for ray tracing!

Photons travel a distance through each region that is proportional to its proper volume

$$\frac{a_I \chi_I}{a_{II} \chi_{II}} = \frac{V_I}{V_{II}}$$

Distance-redshift relation

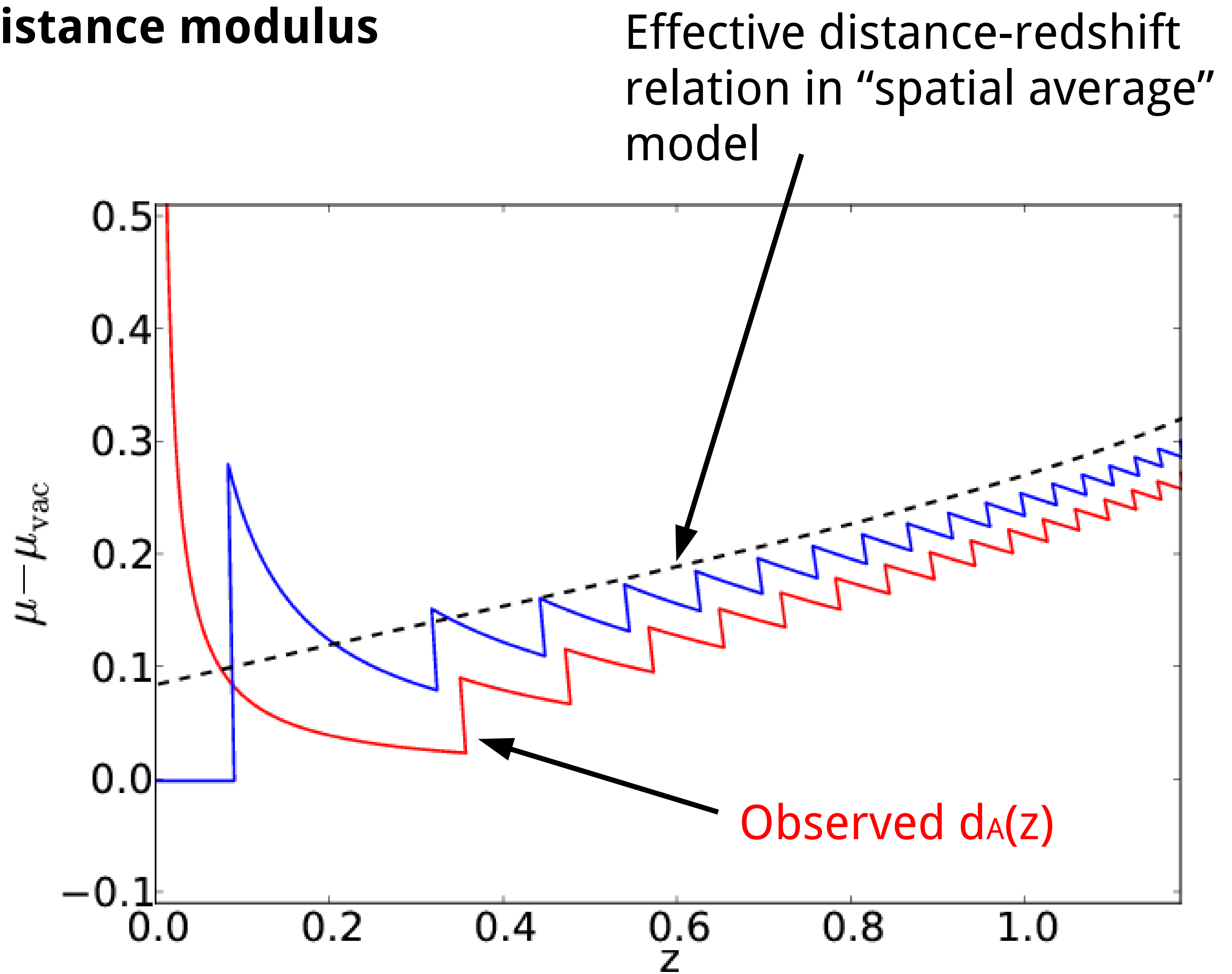
Plot **distance modulus** (log $d_L(z)$ normalised to vacuum)

Flat in vacuum region

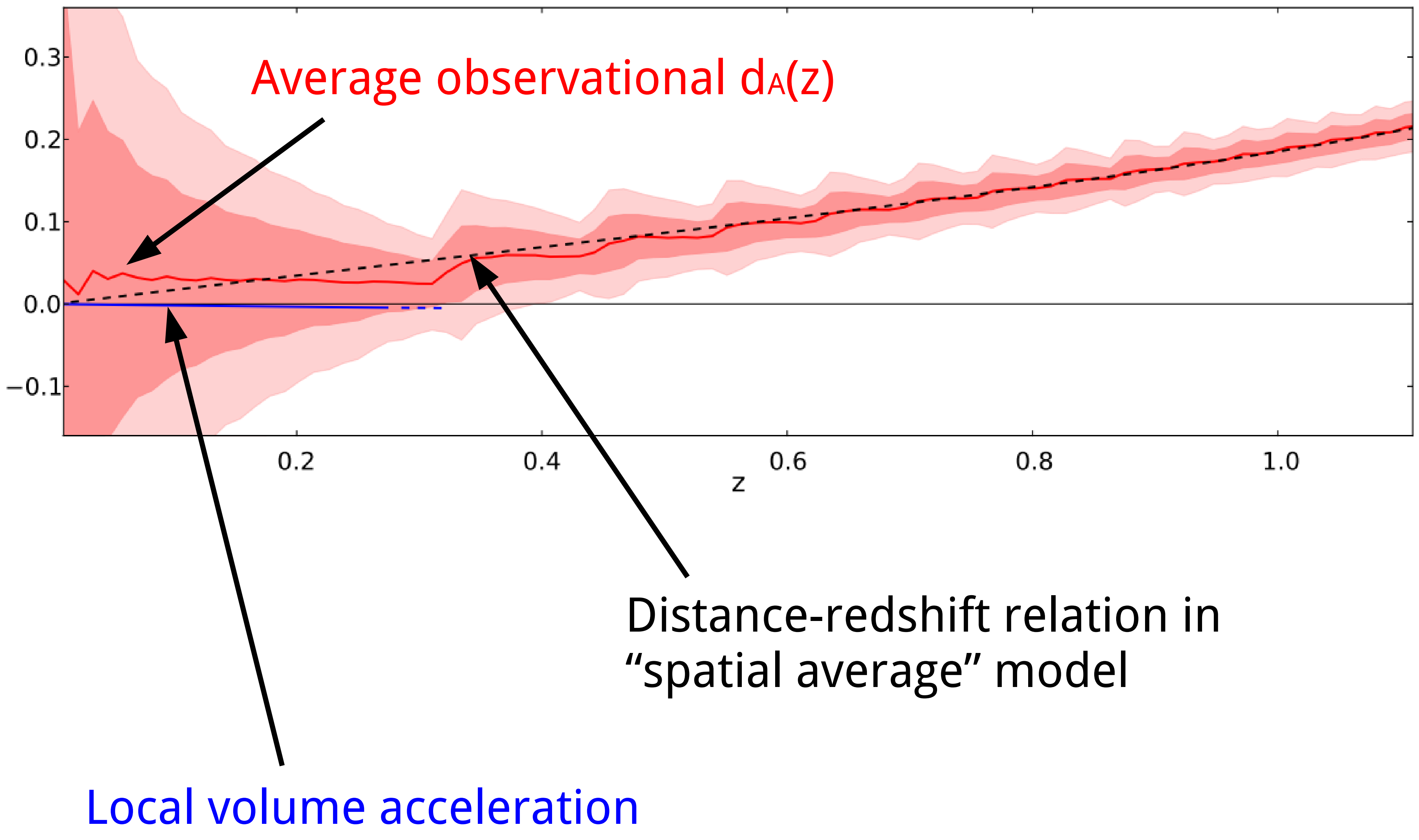
Positive in accelerating expanding region

Negative in decelerating expanding region

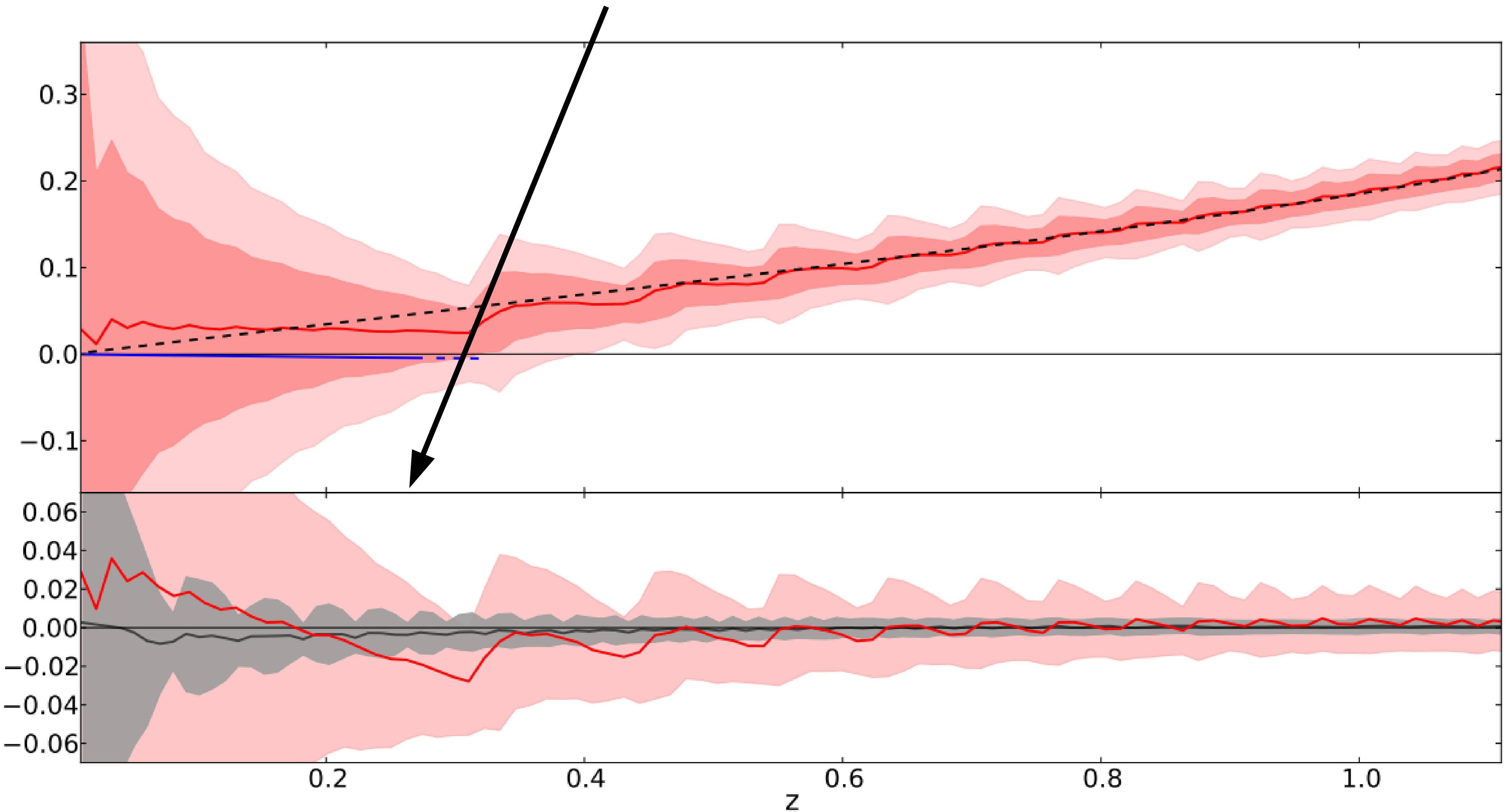
Distance modulus



Distance modulus (relative to vacuum)



Distance modulus (relative to “spatial average”)
(Grey: Smaller FLRW region sizes)



Jagged average is an artefact

Is the model reasonable?

Not a continuous solution to Field Equations

Arbitrary "arrangement" of regions

Note: Spatial average has **not** been fit to average observational relation

Kasner-EdS model

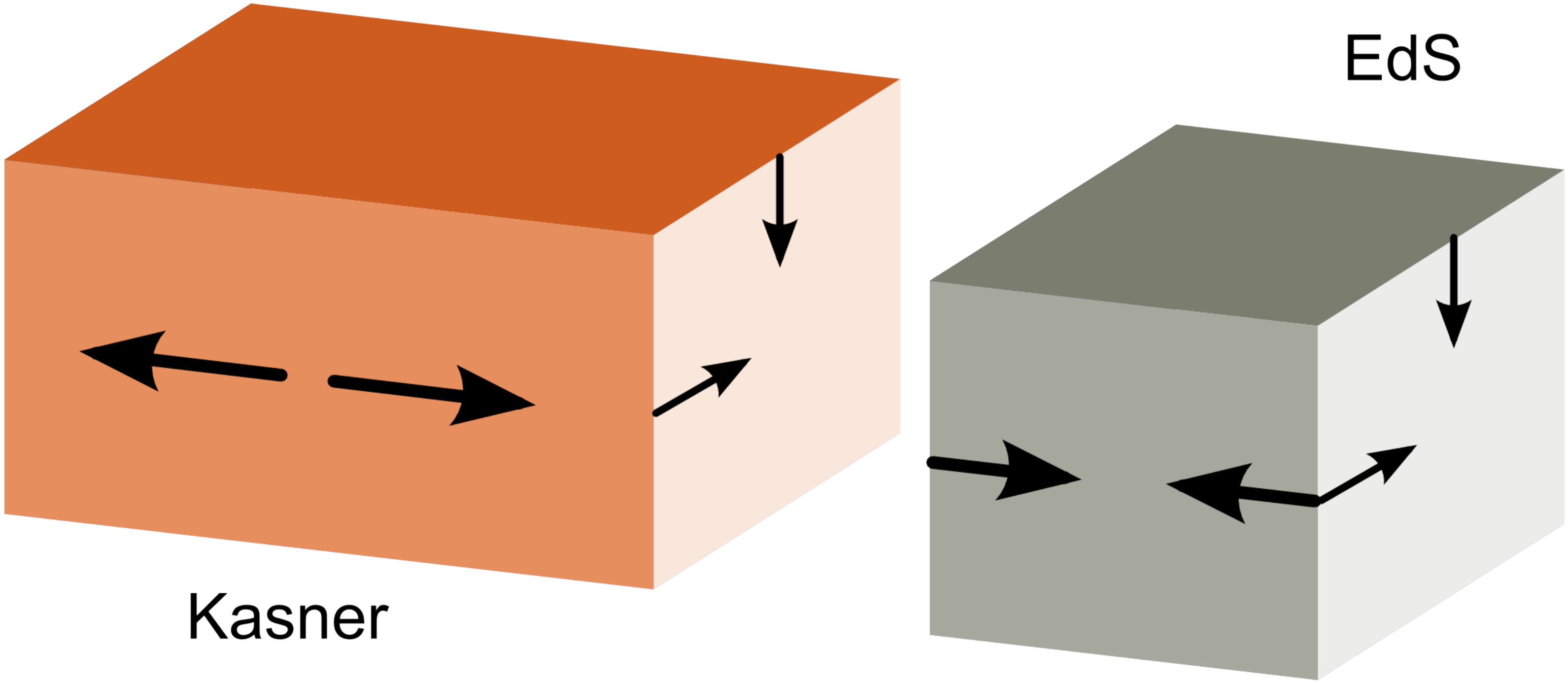
Kasner: anisotropic vacuum (plane-parallel)

$$ds^2 = -d\hat{t}^2 + b_1^2(\hat{t})d\hat{X}^2 + b_2^2(\hat{t})\left(d\hat{Y}^2 + d\hat{Z}^2\right)$$

Can match to collapsing FLRW dust region

Exact solution to Einstein Field Equations

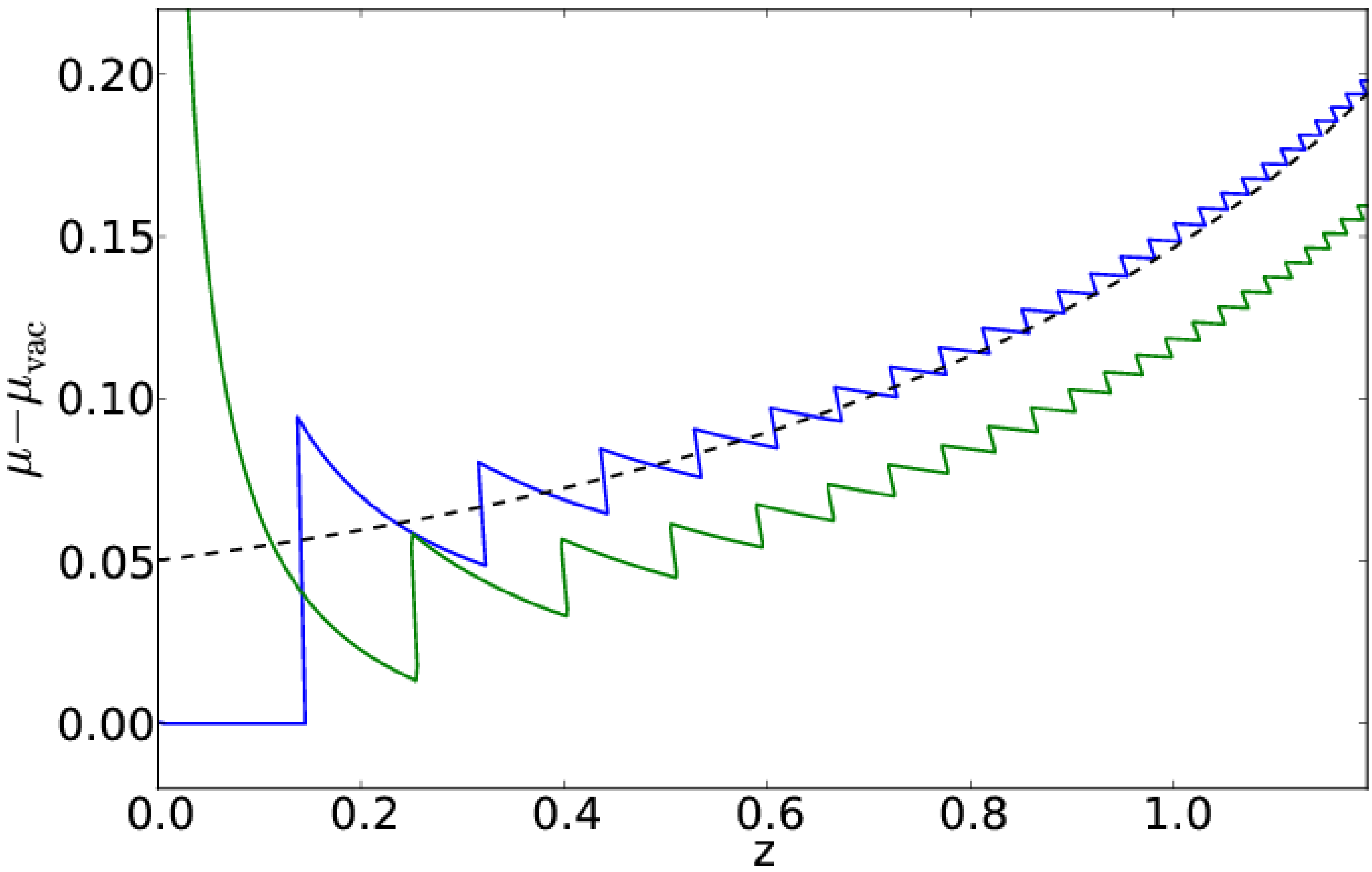
Planar symmetry



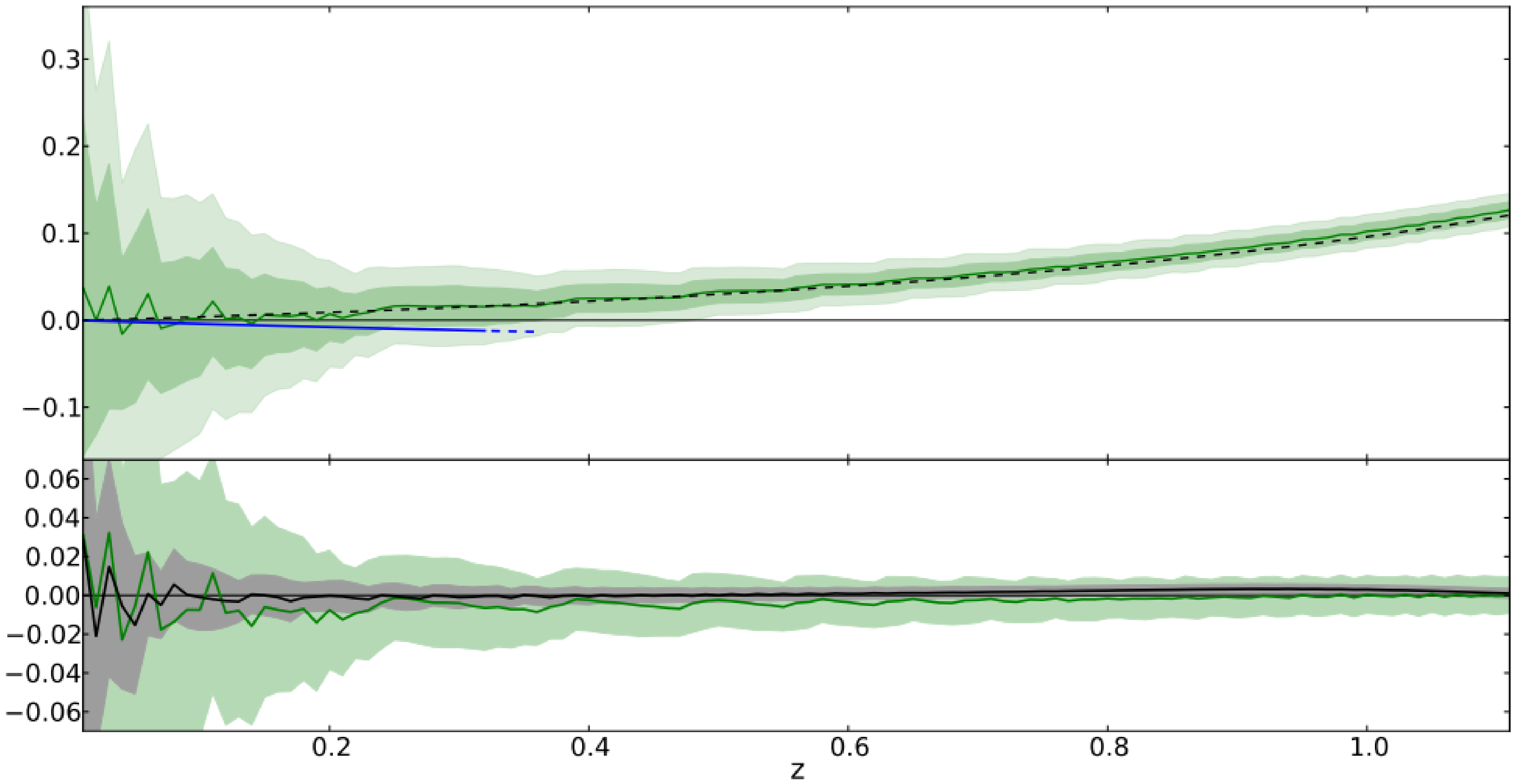
Only concerned with direction orthogonal to matching plane

Define 1D average along this direction

Distance modulus



Kasner-EdS distance modulus curves



Summary

Statistically homogeneous models:

Spatial average model matches model inferred from **observations**

Neither bear much relation to behaviour of **local spacetime**

Inhomogeneous models

Giant void models

Lemaitre-Tolman-Bondi

Spherically-symmetric, inhomogeneous, dust-only spacetime

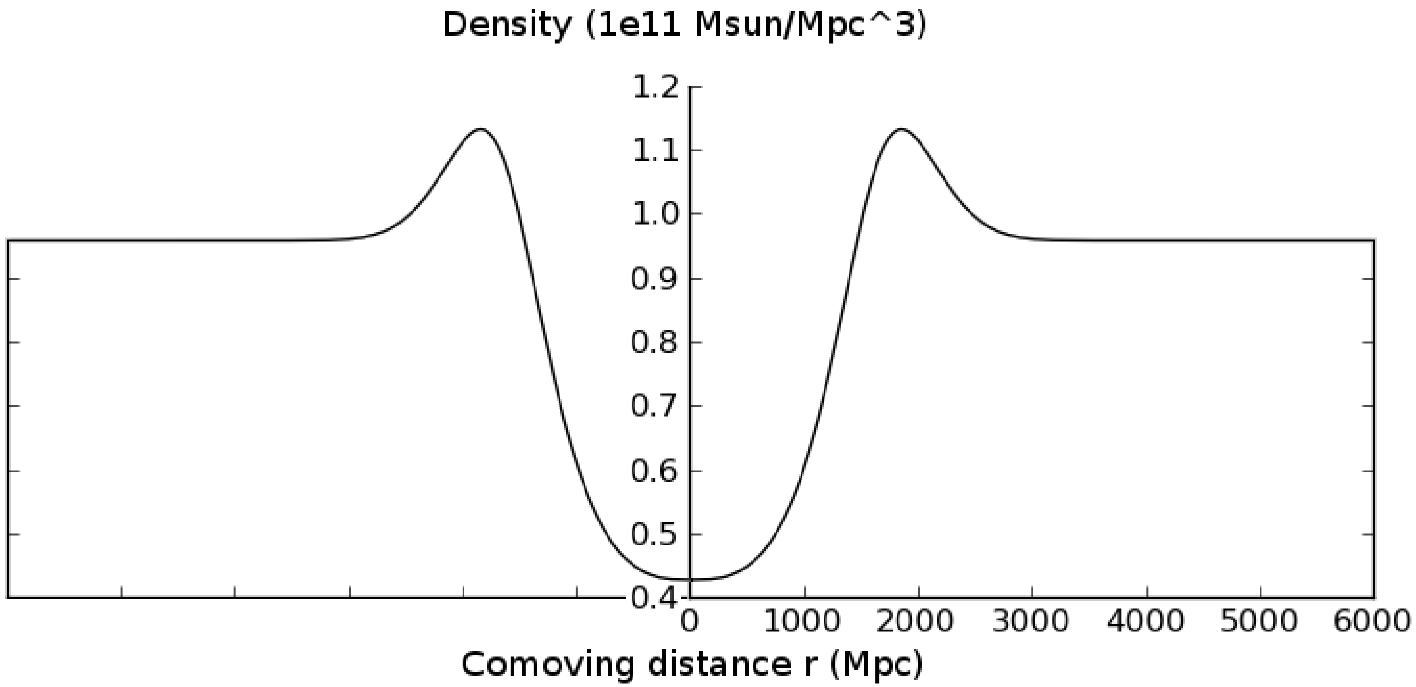
$$ds^2 = dt^2 - \frac{a_2^2(t,r)}{(1 - k(r)r^2)}dr^2 - a_1^2(t,r)r^2 d\Omega^2$$

Isotropic about central observer

Analytic solutions

Giant void models

Hubble-scale underdensity reproduces ΛCDM distance-redshift relation



Alternative model for dark energy

(But now ruled out, see e.g. Bull, Clifton & Ferreira 2012)

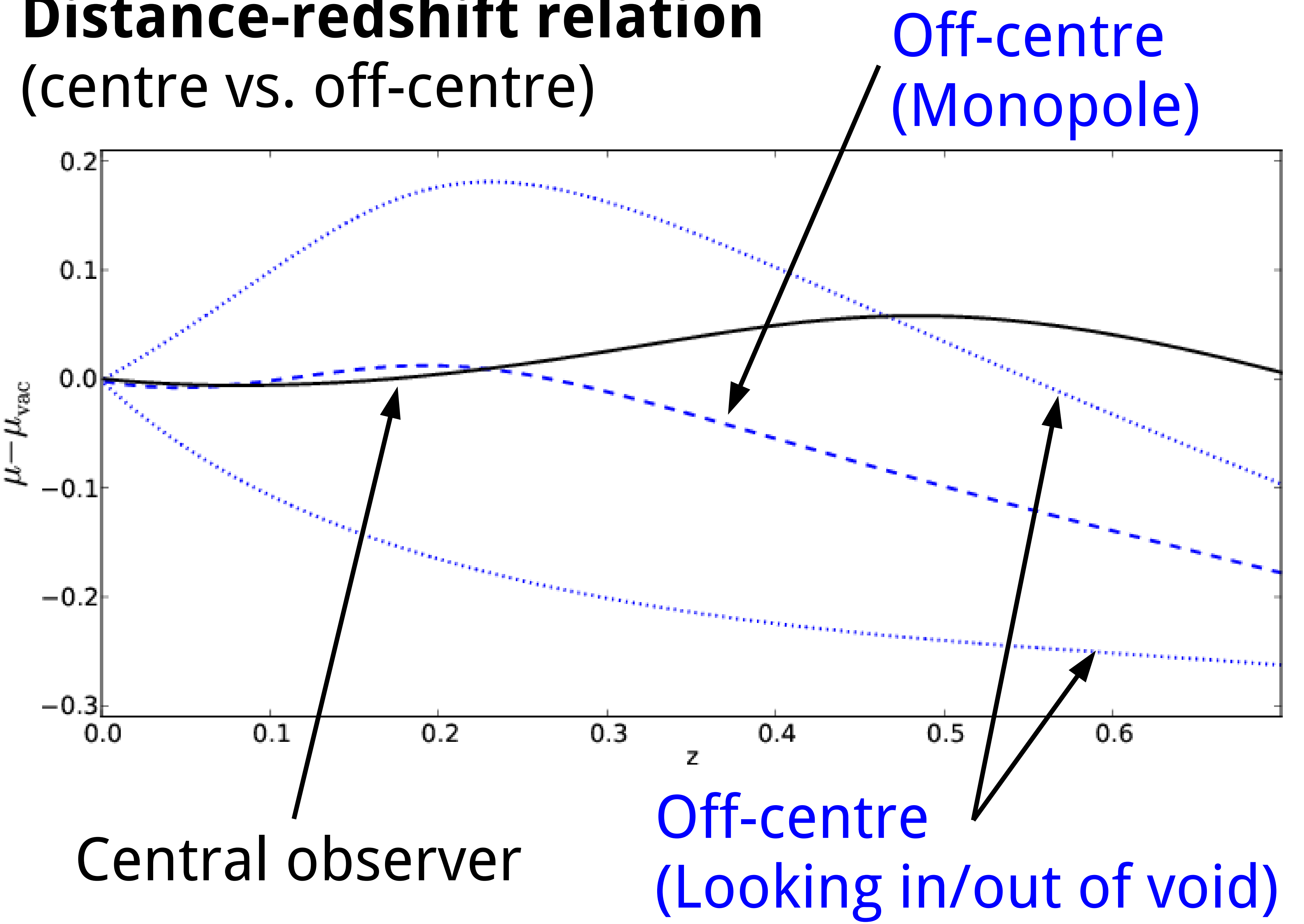
Isotropy and homogeneity

Isotropic only about central observer

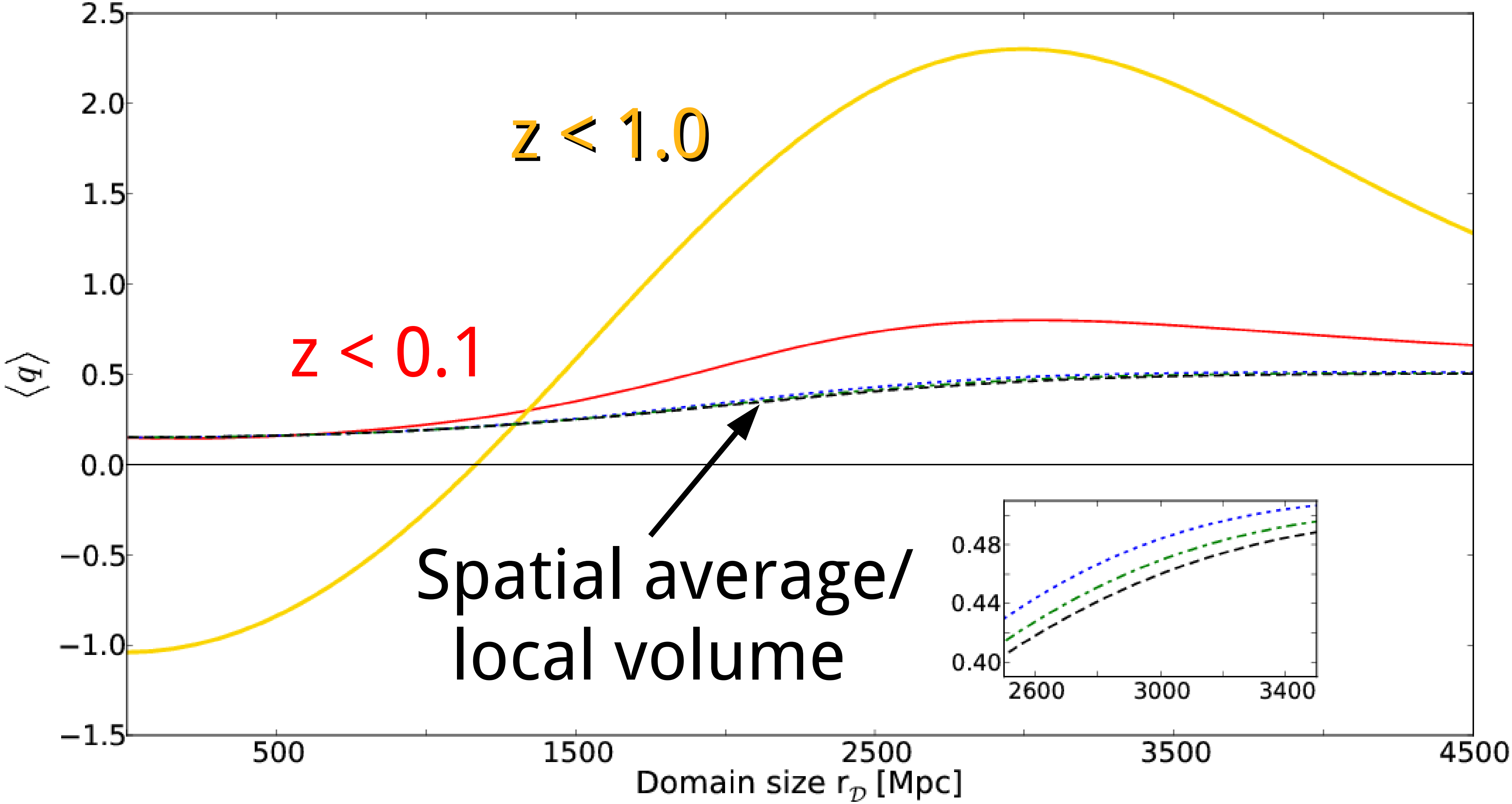
Not statistically homogeneous

No natural choice of spatial averaging domain

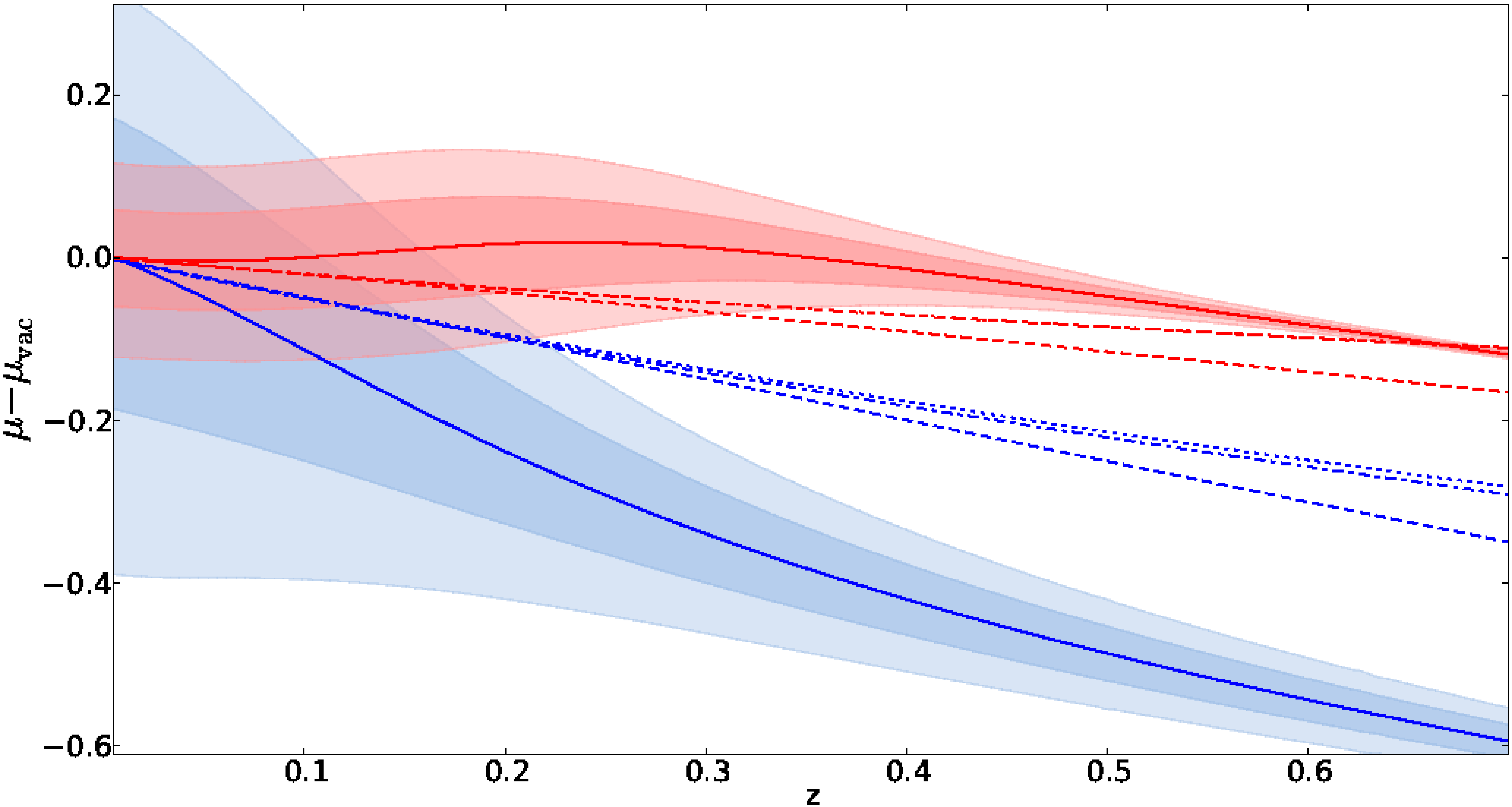
Distance-redshift relation
(centre vs. off-centre)



Deceleration parameters



Distance modulus (vs. averaging domain)



r_D = 1000 Mpc / 3000 Mpc

Summary

Giant void models:

No sensible averaging scale; sensitive to arbitrary choice

Spatial average / local volume acceleration bear little relation to observations

Conclusions

Conclusions

Fitting a homogeneous model to the real, lumpy Universe is an ambiguous procedure

Can define several different “types” of acceleration in general

Acceleration inferred from observations need not correspond to local acceleration of spacetime

P. Bull & T. Clifton, PRD **85** 103512 (2012); arXiv:1203.4479